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# Scattering of sound
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# Outline

- Scattering of single sphere (Cobbold ch 5.1-5.3)
  - Rigid sphere
  - Compressible sphere
- Scattering from a distribution of scatterers ch 5.9
- Scattering from inhomogeneities ch 5.9

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# Scattering cross section

$$\sigma_s = \frac{W_s}{I}$$ scattering cross-section

$$\sigma_d(\theta,\varphi) = \frac{dW}{d\Omega}\frac{1}{I} = \frac{d\sigma_s}{d\Omega}$$ differential scattering cross-section

$$\sigma_b = \sigma_d(\pi, 0)$$ back scattering cross-section

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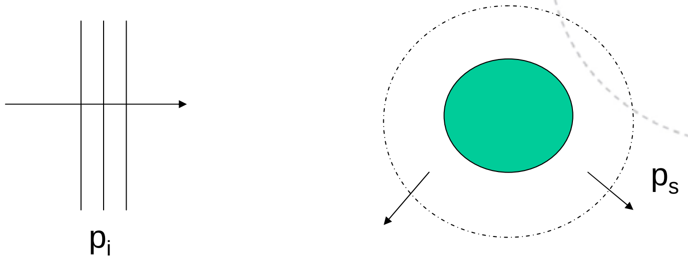
# Scattering by a sphere




$p = p_i + p_s$

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$$\nabla^2\tilde{p} + k^2\tilde{p} = 0$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\tilde{p}}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\tilde{p}}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2\tilde{p}}{\partial\varphi^2} + k^2\tilde{p} = 0$$

$$\tilde{p}_s(r,\theta) = R(r)P(\theta) = \sum_{m=0}^{\infty} A_m P_m(\cos\theta)\, h_m(kr)$$

$$\tilde{p}_i = Ae^{-jkr\cos\theta} = A\sum_{m=0}^{\infty}(2m+1)j^m P_m(\cos\theta)\, j_m(kr)$$

$$h_m(kr) = j_m(kr) - jn_m(kr) = \sqrt{\frac{\pi}{2kr}}\left(J_{m+0.5}(kr) - jN_{m+0.5}(kr)\right)$$

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## The rigid, heavy sphere.

$$\left.\frac{\partial p}{\partial r}\right|_{r=a} = 0 \quad \text{giving}$$

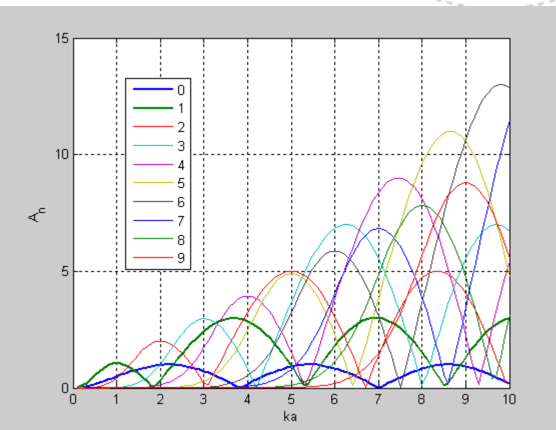
$$A_m = p_0\frac{(-j)^{m+2}(2m+1)}{1+jK_m(ka)} \quad \text{where } K_m(ka) = \frac{\frac{m}{ka}n_m(ka) - n_{m+1}(ka)}{j_{m+1}(ka) - \frac{m}{ka}j_m(ka)}$$

let $kr >> 1$ and $ka << 1$

$$\tilde{p}_s \approx -p_0\frac{k^2a^3}{3r}e^{-jkr}\left(1-\frac{3}{2}\cos\theta\right)$$

$$I_s \approx p_o^2\frac{k^4a^6}{18\rho_o c_o r^2}\left(1-\frac{3}{2}\cos\theta\right)^2$$

$$\sigma_s \approx \frac{7\pi k^4a^6}{9}$$



From cobbold:

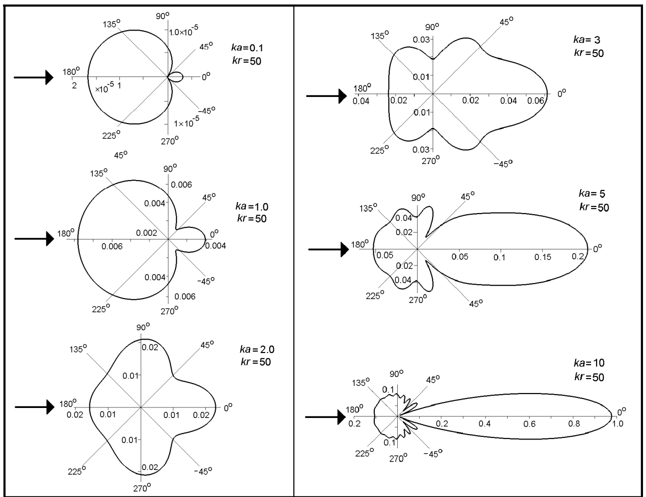



**Fig. 5.2.** Polar plots illustrating the scattering of a plane incident wave (direction given by arrow) by a small rigid sphere that is fixed in space. The magnitudes of the ratio of the scattered to the incident pressure for six different values of *ka* are shown: the numerical values of the ratios are given on the horizontal and vertical axes.

$$\sigma_s \to 2\pi a^2, \quad \text{when } ka \to \infty$$

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Let the sphere be compressible with density $\rho_v$ and speed of sound $c_v$. The surrounding medium has density $\rho_o$ and speed of sound $c_o$.

$$\tilde{p}_v(r,\theta) = \sum_{m=0}^{\infty} B_m P_m(\cos\theta)\, j_m(k_v r)$$

$$\tilde{p}_s(r,\theta) = \sum_{m=0}^{\infty} A_m P_m(\cos\theta)\, h_m(k_o r)$$

$$\tilde{p}_i(k_o a^+) + \tilde{p}_s(k_o a^+) = \tilde{p}_v(k_v a^-)$$

$$\tilde{u}_i(k_o a^+) + \tilde{u}_s(k_o a^+) = \tilde{u}_v(k_v a^-)$$

$$\Downarrow$$

$$\frac{1}{\rho_o}\left[\frac{\partial \tilde{p}_i}{\partial r} + \frac{\partial \tilde{p}_s}{\partial r}\right]_{k_o r = k_o a^+} = \frac{1}{\rho_v}\left[\frac{\partial \tilde{p}_v}{\partial r}\right]_{k_o r = k_o a^-}$$

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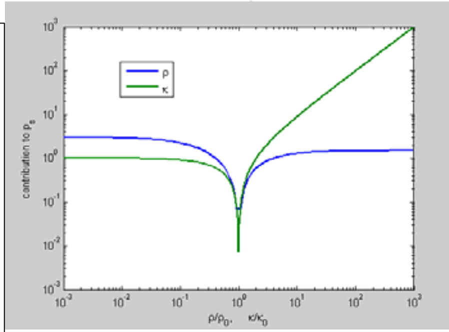
$$A_m = (-j)^m (2m+1) \frac{\rho_v c_v j_m^v j_m'^o - \rho_o c_o j_m^o j_m'^v}{\rho_o c_o h_m^o j_m'^v - \rho_v c_v j_m^v h_m'^o}$$

let kr >> 1 and ka << 1

$$\tilde{p}_s \approx -p_0 \frac{k^2 a^3}{3r} e^{-jkr} \left( \frac{\kappa_v - \kappa_o}{\kappa_o} - \frac{3(\rho_v - \rho_o)}{2\rho_v + \rho_o} \cos\theta \right)$$

$$\sigma_s \approx \frac{4\pi k^4 a^6}{9} \left( \left| \frac{\kappa_v - \kappa_o}{\kappa_o} \right|^2 - \left| \frac{3(\rho_v - \rho_o)}{2\rho_v + \rho_o} \right|^2 \right)$$

$$\sigma_d(\theta) \approx \frac{k^4 a^6}{9} \left( \frac{\kappa_v - \kappa_o}{\kappa_o} - \frac{3(\rho_v - \rho_o)}{2\rho_v + \rho_o} \cos\theta \right)^2$$




From Lars Hoff

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From Cobbold:

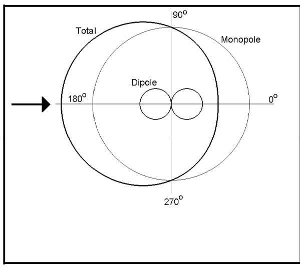


**Fig. 5.3.** Scattering contributions for an RBC in plasma insonated by a plane wave showing the normalized pressure distribution ($|p| \propto \sqrt{\sigma_d}$). The RBC was modeled as a sphere. The dipole and monopole contributions to the total pressure were calculated from (5.19) using the following values for horse blood RBC's in plasma measured by Urick [5.13]: $\rho_o$=1021 kg/m$^3$, $\rho_v$=1091 kg/m$^3$, $\kappa_o$= 0.409 GPa$^{-1}$, $\kappa_v$= 0.341 GPa$^{-1}$, together with $ka$ = 0.1, $kr$ = 50. If the RBC equivalent radius is taken to be 2.7 μm (volume = 82 μm$^3$) then $ka$ = 0.1 corresponds to a frequency of ~8.8 MHz.

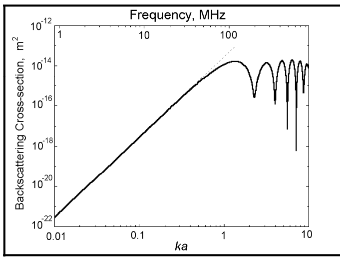


**Fig. 5.4.** Backscattering cross-section for a 2.7 μm radius compressible 'RBC' sphere in saline over a wide range of *ka* values using (5.18) with 20 terms in the expansion (no change occurred with further terms). The dashed line shows the results given by the approximate (small *ka*) equation which corresponds to an $f^4$ frequency dependence. The following parameters were used: $\rho_v$=1078 kg/m$^3$, $\kappa_v$= 0.39115 GPa$^{-1}$, $\rho_o$=1004 kg/m$^3$, $\kappa_o$=0.44206 GPa$^{-1}$, $kr$=50, $a$=2.7 μm.

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# Distribution of scatterers

- Common to use Rayleigh scatterers with distributed strength and/or posistion (blood, liver tissue)
- Distribution of scatterers presented in e.g.Angelsen ch.7.4-5 and Cobbold ch.5.9

- Stochastic models (gaussion distributions, correlation)